Dynamic Characteristics of Statically Determinate Space-Truss Platforms

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The geometry of a class of statically determinate platforms is developed and vibration frequencies determined. Such configurations allow shape control by changing the member lengths to be accomplished with small forces. An additional advantage of a statically determinate structure is that it is free of thermal stress under any temperature distribution. Frequency comparisons between statically determinate and more conventional redundant platforms are presented. Vibration of curved platforms that can be used as antenna concepts is also investigated. Alternative concepts incorporating the statically determinate design, but having improved dynamic characteristics, are suggested.

Nomenclature

D = width of platform
E = Young's modulus of material
f = frequency of vibration
F = focal length of parabolic surface platform
H = depth of platform
J = number of joints in platform
M = number of members in platform
w = mass per unit length of member
W = concentrated mass at a joint
ρ = mass density of material

Introduction

ACCURACY requirements envisioned for future large space antenna structures may require active control or static adjustment of the surface. One means to accomplish such control is by changing lengths of individual members of a supporting truss structure. This is most easily accomplished if the supporting structure is statically determinate so that essentially no force is required to change the shape of the surface. A statically determinate structure also has the advantage of being free of thermal stress for all temperature distributions. This may be of special importance for configurations having very slender flexible members that can support only small compressive loads without buckling. Haftka and Adelman[1] proposed a concept that utilizes controlled temperature changes in selected members to effect changes in the member lengths in order to minimize surface errors. A question arises as to the structural performance of a large statically determinate platform that might be used to support an antenna surface. In this paper, general configurations that result in a statically determinate rectangular platform structure are devised. The vibration modes and frequencies of such structures are determined and compared to those of a redundant platform having the same size and joint arrangement. A measure of the relative stiffness of the statically determinate platform is thus obtained.

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Geometry of Statically Determinate Platforms

A statically determinate pin-connected space truss must satisfy the equation

$$M = 3J - 6 \tag{1}$$

where M is the number of members and J the number of joints. However, satisfaction of Eq. (1) is only a necessary condition; parts of the structure could be mechanisms, while other parts could be redundant. One approch to ensuring a structure without singularities is to start with a statically determinate truss and add three members and a joint by connecting the new members to any three existing joints that do not lie on a straight line. This procedure can be repeated as many times as necessary and at each step the configuration is a stable, statically determinate structure. Application of this procedure to generating a statically determinate platform is illustrated in Fig. 1.

The starting point is the cube shown in Fig. 1a having one diagonal in each face. This configuration has 8 joints and 18 members that satisfy Eq. (1) and is obviously not a mechanism. By adding one node and three members at a time, the resulting structure remains statically determinate. An additional cube along side the original cube can be constructed in this manner, as illustrated in the successive steps of Figs. 1b-1e. The same process can be used to construct a cube attached to an adjacent surface of the central cube as shown in Fig. 1f. The corner can be filled in with two nodes and six members, as shown by Figs. 1g and 1h.

This process could be repeated in all four directions to generate any size of rectangular statically determinate platform. The resulting configuration would appear as shown in Fig. 2. There is a central cross formed by the intersection of two stiff, square-cross-section beams with each of the four quadrants filled in by cubes that would by themselves be mechanisms. All of the frames running in the horizontal direction have diagonal stiffening, as depicted by the lower view in the figure. The frames running in the vertical direction (except for the one marked with an asterisk) do not have diagonal stiffening, as shown by the view on the right. Several variations to the construction of the cubes in each quadrant are possible; the one shown has members along all six edges, but diagonal stiffening is present in only one direction.

Analysis

Vibration modes and frequencies were determined using the MSC/NASTRAN[2] and EAL[3] programs. All members were assumed to be pin connected at all joints. Preliminary calculations were made using the BUNVIS[4] program to analyze the two-bay platform shown in Fig. 1h. (All of the platforms examined in this paper were square with an equal number of bays on each side.) The structure was modeled using beam members pinned at each end and having bending and twisting stiffnesses large enough to insure that the local pin-ended modes had frequencies well above that of the overall platform frequency. The results of this analysis using BUNVIS[4] are exact in the context of beam column theory representing each individual member of the structure.

Comparison of these frequency calculations with NASTRAN[2] and EAL[3] established that conventional finite element programs gave accurate results with nodes only at the joints and provided a consistent mass matrix was used. This could be accomplished in EAL with rod elements having axial stiffness only. However, in NASTRAN, when a consistent mass matrix is specified for the rod elements, only the terms associated with axial motion of the rod are included; these terms are not important for the configurations discussed in this paper.

For the lateral motion of the member, the lumped mass matrix is retained, leading to significant errors. The problem was overcome by using beam elements with the end moments removed by the use of the pin-flag option. Having established a modeling procedure that gives accurate results for two bays, results for a greater number of bays are expected to be accurate since the accuracy of conventional finite element analyses improves with the increasing number of bays. Frequencies and modes determined from NASTRAN and EAL were essentially identical for all of the configurations subsequently analyzed.

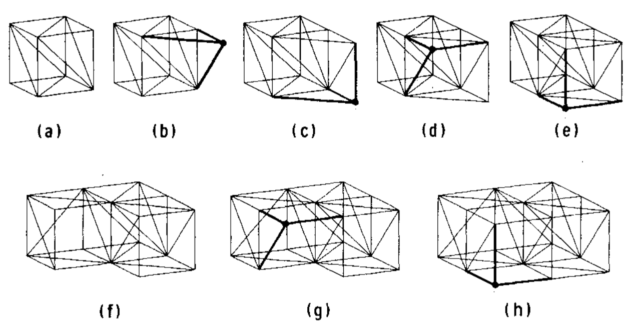


Fig. 1 Generation sequence for a statically determinate platform ($M = 3J - 6$).

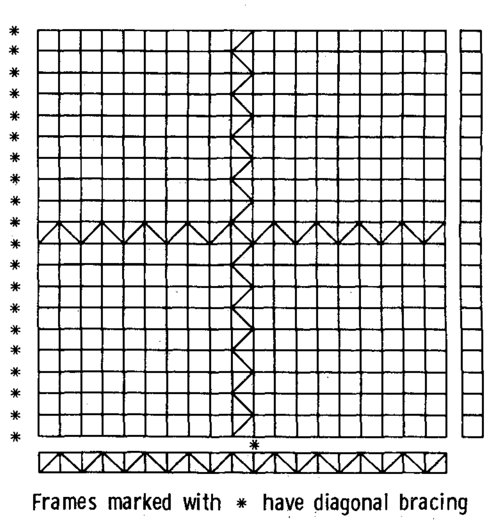


Fig. 2 Statically determinate platform configuration 1.

Configurations

All of the platforms analyzed had an equal number of bays in both directions with a depth equal to bay width. All members had the same cross-sectional area. The four statically determinate configurations shown in Figs. 2 and 3 were analyzed. The rationale for this selection and a detailed description of the three additional configurations shown in Fig. 3 are given in the section on results. Two additional configurations that were structurally redundant were analzyed: 1) a fully stiffened configuration having the same joint locations as the statically determinate configurations, but with diagonal members in every square panel; and 2) a partially stiffened configuration obtained by adding diagonal members to all core panels of a statically determinate configuration. Since these configurations have different member lengths and, in the case of the redundant configurations, different number of members, their masses are not the same for the same size platform. In addition, some cases were analyzed with an additional mass W at each node that was five times the mass of a vertical member. Table 1 gives a comparison of the mass of the various configurations relative to the statically determinant configuration 1 without nodal masses for the case when the number of bays becomes very large.

Results and Discussion

The lowest vibration mode for statically determinant configuration 1 of Fig. 2 and the corresponding fully stiffened

Table 1 Mass characteristics for configurations analyzed

Configuration	Mass ratio W/Hw 0	Mass ratio W/Hw 5
1	1.00	2.56
2	1.05	2.61
3	1.00	2.56
4	1.06	2.62
Fully stiffened	1.66	3.22
Partially stiffened	1.22	2.78

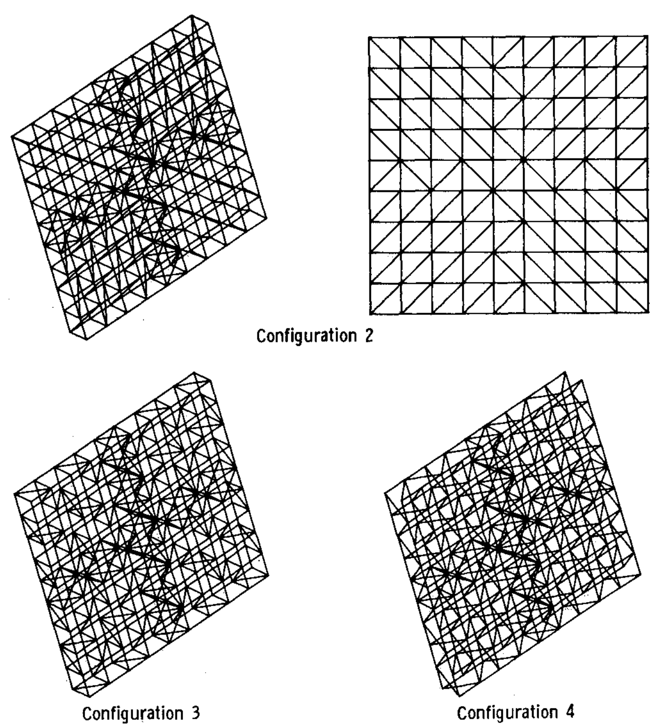


Fig. 3 Alternate statically determinate configurations.

configuration is shown in Fig. 4. The nondimensional frequency parameter shown is based on considering the platform as an equivalent flat plate. The fully stiffened configuration exhibits modes similar to those found for a free flat plate and its frequency is over an order of magnitude greater than the statically determinate configuration. It can be seen in Fig. 4 that the diagonal stiffening is in only one direction in each of the four quadrants for the statically determinate configuration. Thus, configuration 1 has a preferred direction for resisting the bending loads. Normal to this direction, the transverse shear stiffness is small. Lines running in the stiff direction remain essentially straight and only rotate; the largest curvature is normal to the stiff direction.

Alternate Statically Determinate Configurations

The mode shape shown in Fig. 4 for configuration 1 suggests that an increase in frequency might be possible by an arrangement that provides stiffness in two directions. To provide stiffness in both directions, the configuations of Fig. 3 were devised. All of these configurations start with the same basic central cross that was developed in Figs. 1 and 2. The various configurations result from the particular manner in which the individual cubes in each quadrant away from the central cross are constructed. Configuration 2 is obtained by spanning both in-plane directions of the platform with one diagonal member in each cube. These diagonals are symmetrically arranged as indicated in the planform view of configuration 2. The planform of all the other configurations is the same as indicated in Fig. 2. Configuration 3 is obtained by alternating the direction of the diagonal members. Configuration 4 is obtained by removing the vertical members, which allows diagonal members in both directions. Comparison of the lowest frequencies of these configurations with that of configuration 1, the statically determinate platform of Fig. 2, is shown in Fig. 5 for platforms having five and nine bays. Compared to configuration 1, a substantial increase in frequency is obtained for all of the additional configurations. Because configuration 2 had the greatest increase, it was used for more extensive calculations that are given in the remaining sections of this paper. However, calculations made for a larger number of bays than show in Fig. 5 show that configurations 2 and 3 tend to have the same frequencies.

Frequency Comparison for Statically Determinate and Fully Stiffened Platforms

In Fig. 6, the lowest three frequencies are plotted vs the number of bays on a side for both the statically determinate configuration 2 and the fully stiffened platform. Figure 6a applies to a platform with no additional nodal masses. The value of $W/Hw=5$, used in Fig. 6b, is such that the total added mass at the joints is about 1.5 times the mass of the statically determinate structure (see Table 1). The nondimensional frequency parameter should be constant for each mode, based on

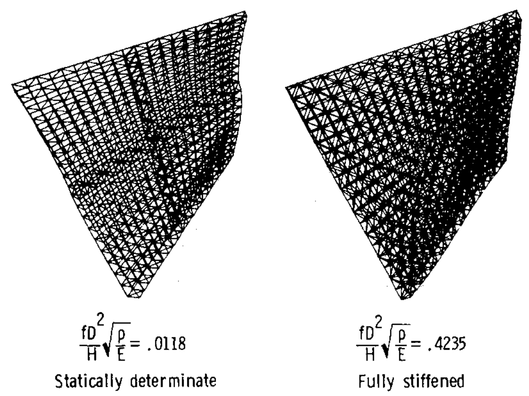


Fig. 4 Comparison of modes and frequencies for statically determinate and fully stiffened platforms.

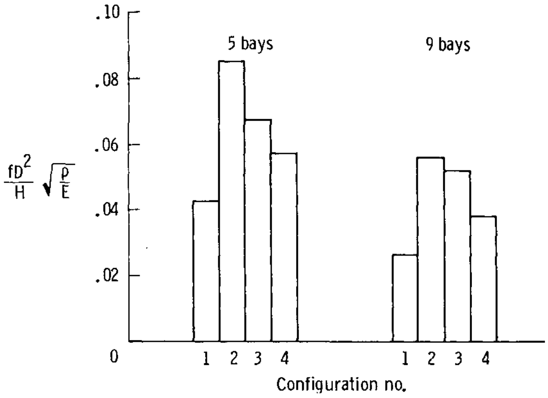
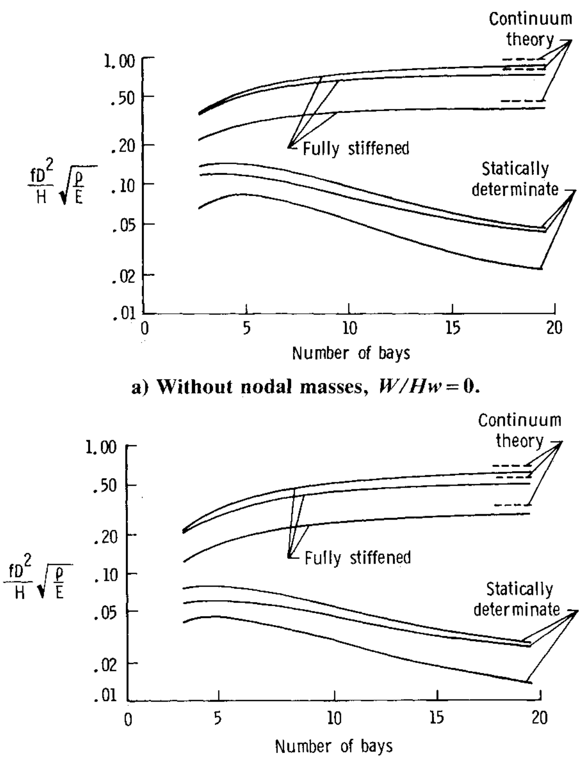


Fig. 5 Effect of configuration on frequency.



a) Without nodal masses, $W/Hw=0$.

b) With nodal masses, $W/Hw=5$.

Fig. 6 Frequency of statically determinate and fully stiffened platforms.

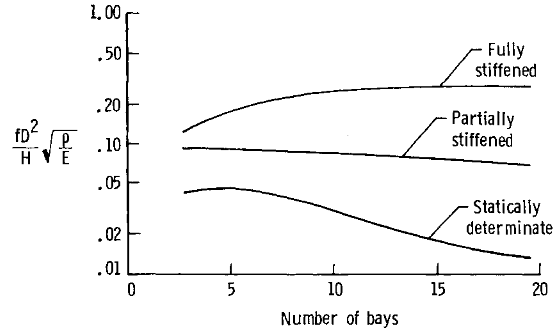


Fig. 7 Frequency of platform with various degrees of stiffening, $W/Hw=5$.

the continuum bending stiffness of the platform from Noor et al.[5] The fully stiffened design corresponds to the hexahedral platform of Ref. 5 and the frequencies are in good agreement with the continuum theory for a large number of bays. The small differences in frequencies for a high number of bays are attributed to rotary inertia effects, while the large differences for platforms with a small number of bays is due to tranverse shear effects. Frequencies for the statically determinate configuration decrease with the increasing number of bays and are far below the results for the fully stiffened configuration.

Additional frequency calculations were made for a partially stiffened configuration. This configuration has full diagonal stiffening (as shown in the lower part of Fig. 2) in every core frame, but the faces remain the same as in the planform view of Fig. 2. The lowest frequency is shown in Fig. 7 for the statically determinant, partially stiffened and fully stiffened platforms. Despite a significant increase in the number of members, the partially stiffened configuration does not develop a plate twisting stiffness; its frequency is far below that of the fully stiffened configuration and the difference increases with number of bays. It appears impossible to find any configuration that can develop all of the necessary plate stiffnesses comparable to the fully stiffened configuration and remain statically determinate. Equation (1) allows only six members at a typical interior node, while it appears that an average of nine members per interior node as found in the fully stiffened configuration is required to develop a fully effective platelike platform.

Vibration of Curved Platforms

A space antenna is a prime candidate for a structure that might utilize the advantages of a statically determinate structure for contorl of the surface geometry. Parabolic-shaped platforms were analyzed to determine if there were any major differences in the frequency characteristics compared to flat platforms. Configuration 2 with an F/D (focal length-to-diameter ratio) of 0.5 is shown in Fig. 8. Results from Hedgepeth and Adams[6] show that the frequency of a shallow spherical cap with free edges is little different from a flat circular plate. Calculations of the effect of curvature on the vibration of the configurations of this report agreed with Ref. 6 in showing essentially no change in frequency with curvature for values of $F/D \geq 0.5$.

Application of Results

Although the frequency of a statically determinate platform is far below a fully stiffened platform, it still may be sufficient for some applications. Calculations have been made of the maximum platform size as a function of frequency requirements. The result is shown in Fig. 9 for material properties typical of graphite-epoxy laminate, $E = 110$ GPa and $\rho = 1600$ kg/m^3. The calculations were made from the results shown in Fig. 6b, which included an additional mass at each joint corresponding to $W/Hw = 5$. For a given size and minimum frequency requirement, the statically determinate platform would generally have fewer bays than a fully stiffened platform and, consequently, be deeper and have longer members. Longer members may require larger cross-section dimensions to satisfy the minimum frequency requirements for local member vibration. Despite their reduced performance, statically determinate platforms of practical proportions as large as 100 m can be built to have frequencies up to 0.5 Hz, which may be more than necessary in some applications.

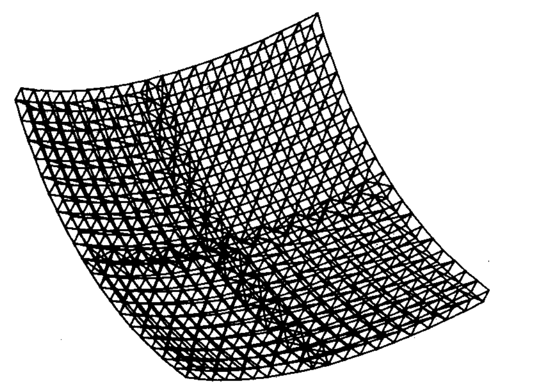


Fig. 8 Generation of curved platform having $F/D = 0.5$.

Modified Concepts

If the frequency performance of the statically determinate concept is not adequate, modifications that retain the advantages of the concept, but with improved dynamic characteristics, may be possible. Consider a statically determinate platform to have special additional members in panels that do not have diagonal members. An example is indicated in Fig. 10, where the unshaded members are a portion of a frame of a statically determinate platform. As shown in the figure, additional diagonal members could provide axial damping or be actively controlled to provide stiffness when required. In the fist case, a damper would be designed to provide a very stiff connection for the frequencies of concern; however, for much smaller frequencies (longer periods), little or no resistance would develop, so that shape changes due to active control or temperature could occur with very small member forces. Such a design could exhibit frequencies at finite amplitudes approaching those of the fully stiffened platform.

An alternative concept would have special members having a locking mechanism that is released when ever control is desired and thus allowing a shape change with negligible force.

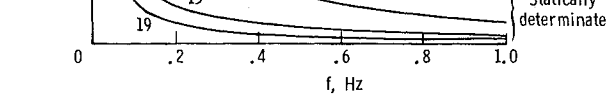


Fig 9 Allowable size of statically determinate and fully stiffened platforms based on minimum frequency requirements, $W/Hw = 5$.

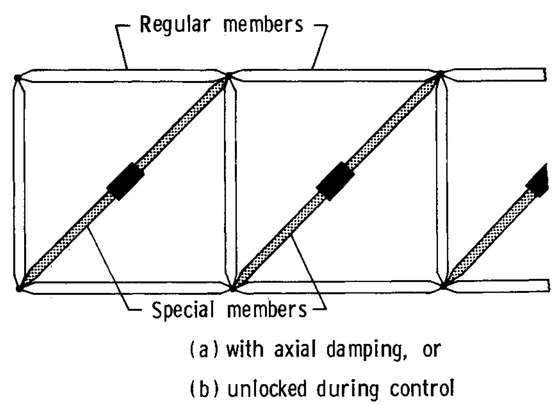


Fig. 10 Concepts for augmenting stiffness and damping of statically determinate platforms.

During periods not being controlled, the members would be locked to give the platform the dynamic characteristics of a fully stiffened configuration.

Conclusions

Structural arrangement for several statically determinate truss platforms have been developed. Such structures would have desirable characteristics for shape control and elimination of thermal stresses. The vibration frequencies of such platforms were found to be substantially lower than a fully stiffened structure. Even though the frequency of a statically determinate platform is lower than a more conventional design, numerical results show that statically determinate platforms as large as 100 m can be designed to have frequencies of up to 0.5 Hz. For improved performance, a modified statically determinate configuration having special additional members is suggested. These additional members have special damping properties or are capable of being released by a control system. The resulting configuration may have the potential of retaining the advantages of the statically determinate platform, while exhibiting the higher dynamic performance of a fully stiffened platform.

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